INVESTIGATING THE THERMAL INTERACTION BETWEEN A GAS WELL AND PERMAFROST SOILS

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We present a method, a description of the algorithm, and the results from the calculation of the thermal fields about a gas well drilled through permafrost soil.

As a gas moves through the shaft of a well drilled into permafrost soil the latter is thawed because the gas temperature in the gas-bearing layers is quite high. In turn, the frozen soil cools the gas, which may result in the formation of crystalline compounds from the gas and the water, leading to the blockage of the well. To choose the optimum operating regimes for the well and to select the optimum designs, we must undertake a simultaneous gas-thermodynamics and thermal calculation of the well–soil system.

These processes are described by a system of differential equations that are rather complex in form. In this case the problem does not reduce to the solution exclusively of the differential Fourier equation with the Stefan conditions, since we must take into consideration the processes occurring within the gas moving through the well. It is therefore more advisable and promising to use electronic digital computers for such calculations, and the computers – because of their universality – enable us to perform the calculation for the well–soil system with simultaneous consideration of all operative factors, and the algorithm realized in the program may include logical operations dealing with the processing of the initial information and the derived results. For problems of this class this latter circumstance is of great significance: first of all, the initial information includes climatological factors which vary according to statistical laws and must be processed accordingly; secondly, for variation calculations of multiyear regimes of gas-well operation we obtain a tremendous amount of results, whose processing and analysis by manual calculation methods requires a substantial expenditure of time.

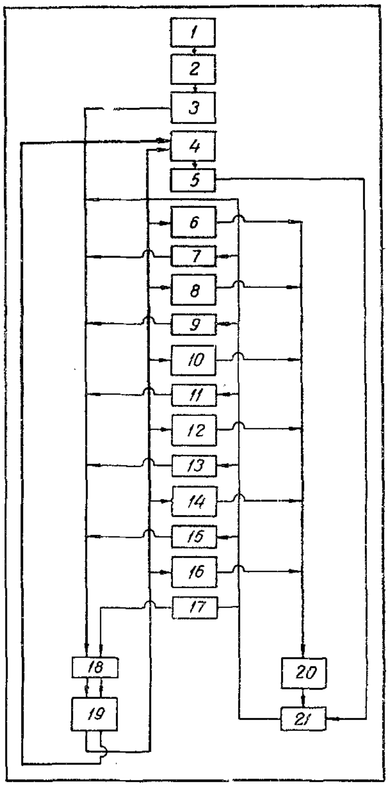

Fig. 1. Block diagram of the program.

Below we discuss the algorithm, the program, and the results from the calculation of the temperature field about a gas well (for the conditions of a single northern deposit).

1. The following information is given: the diameter and depth of the well, the temperature of the gas-bearing layer, the thermophysical characteristics of the soil (variable through the depth), the average monthly temperatures of the ambient air, the duration and average thickness of the snow cover. We have to determine the temperature field of the soil surrounding the well, with the latter in prolonged operation and with a change in the temperature of the gas through the shaft of the well at various gas flow rates.

Since this is an axisymmetric problem, the temperature field can be treated as a two-dimensional field – a variation in the temperature with time, through the height and along the radius, i.e., $t(\tau, x, r)$.

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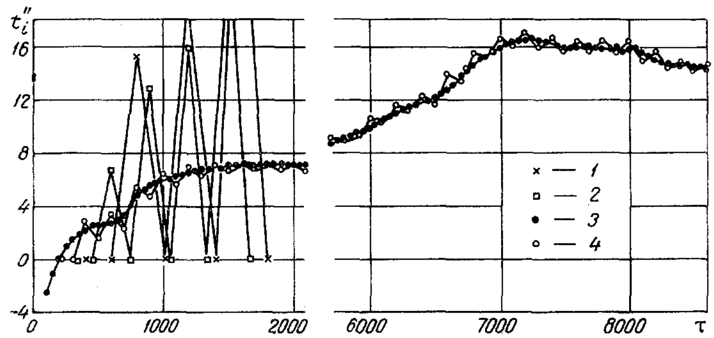

Fig. 2. Convergence of the calculation process for various magnitudes of the theoretical time interval $\Delta\tau$ (t_i'' is the soil temperature, °C; τ is the time from the start of the calculation, the start-up of the well, h): 1) $\Delta\tau$ = 200; 2) 150; 3) 100; 4) 50 h.

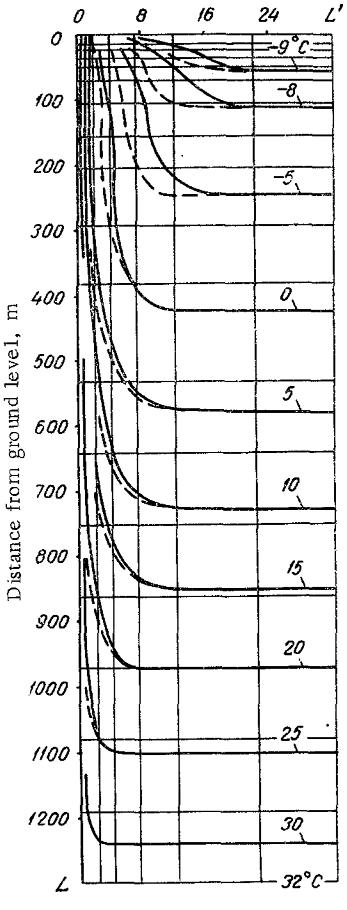

Fig. 3. Isotherms in the soil about the well (L is the distance to ground level, m: L' is the distance to the wall of the well, m); the numerals at the curves denote the temperature values for the soil.

The boundary conditions are the following (Fig. 3): the temperature at the bottom is constant and equal to the temperature of the gas-bearing layer; to the left the temperature of the gas in the well, a function of the gas flow rate, varies with time and through the height, i.e., $t(G, \tau, x)$; at the top, the temperature of the ambient air is variable in time, i.e., $t(\tau)$; to the right, the temperature of the soil unaffected by the well is variable through the height, while in the upper layers of the soil, where seasonal temperature fluctuations are felt, the soil temperature varies with time, i.e., $t(x, \tau)$. The region is divided into theoretical volume blocks on the basis of a nonuniform rectangular grid. Horizontally, we took 8 block columns with widths of 1, 1, 2, 3, 5, 10, and 30 m; vertically, we took 20 block columns with heights of 2, 3, 5, 8, 12, 15, 20, 35 with 50, 2 with 87 and 156 and 7 with 110 m. The dimensions along the vertical were chosen so that a whole number of blocks is associated with each soil layer exhibiting distinct thermophysical characteristics.

2. An explicit scheme is used to compile the system of algebraic equations approximating the partial differential equation describing the process of nonsteady heat transfer in the soil.

Each equation is an expression of the heat balance for the theoretical volume during the time interval $\Delta\tau$. The increment in the enthalpy of the block is equal to the sum of the heat flows coming in from all sides (there are four such sides in a two-dimensional scheme) into this block:

$$C_i\,(t_i''-t_i')=\left[\frac{1}{R_{i-(i-1)}}\,(t_{i-1}'-t_i')+\frac{1}{R_{i-(i+1)}}\,(t_{i+1}'-t_i')+\frac{1}{R_{i-(i-n)}}\,(t_{i-n}'-t_i')+\frac{1}{R_{i-(i+n)}}\,(t_{i+n}'-t_i')\right]\Delta\tau. \tag{1}$$

In Eq. (1)

$$C_i=c_i\gamma_i l_i\pi\,\left(r_{(i+1)b}^2-r_{ib}^2\right),$$

$$R_{i-(i-1)}=\frac{1}{2\pi l_i}\left(\frac{1}{\lambda_i}\ln\frac{r_i}{r_{ib}}+\frac{1}{\lambda_{i-1}}\ln\frac{r_{ib}}{r_{i-1}}\right),$$

$$r_i=\sqrt{\frac{r_{ib}^2+r_{(i+1)b}^2}{2}},\quad r_{i-1}=\sqrt{\frac{r_{(i-1)b}^2+r_{ib}^2}{2}},$$

$$R_{i-(i-n)}=\frac{1}{2\pi\,(r_{(i+1)b}^2-r_{ib}^2)}\left(\frac{l_i}{\lambda_i}+\frac{l_{i-n}}{\lambda_{i-n}}\right);$$

$R_{i-(i+1)}$ and $R_{i-(i-n)}$ are calculated in analogy with $R_{i-(i-1)}$ and $R_{i-(i-n)}$. After substitution into (1) of the values of the quantities in that equation and after performing appropriate elementary transformations, the equation assumes the form

$$t_i'' = t_i' + \frac{\Delta\tau}{c_i\gamma_i l_i (r^2_{(i+1)b} - r^2_{ib})}\left[\frac{2l_i}{\frac{1}{\lambda_i}\ln\frac{\sqrt{0.5(r^2_{ib}+r^2_{(i+1)b})}}{r_{ib}} + \frac{1}{\lambda_{i-1}}\ln\frac{r_{ib}}{\sqrt{0,5(r^2_{ib}+r^2_{(i-1)b})}}}\right.$$

$$\times (t'_{i-1} - t'_i) + \frac{2l_i}{\frac{1}{\lambda_i}\ln\frac{r_{(i+1)b}}{\sqrt{0.5(r^2_{ib}+r^2_{(i+1)b})}} + \frac{1}{\lambda_{i+1}}\ln\frac{\sqrt{0.5(r^2_{(i+1)b}+r^2_{(i+2)b})}}{r_{(i+1)b}}}$$

$$\left.\times (t'_{i+1} - t'_i) + \frac{2(r^2_{(i+1)b} - r^2_{ib})}{\frac{l_i}{\lambda_i} + \frac{l_{i-n}}{\lambda_{i-n}}}(t'_{i-n} - t'_n) + \frac{2(r^2_{(i+1)b} - r^2_{ib})}{\frac{l_i}{\lambda_i} + \frac{l_{i+n}}{\lambda_{i+n}}}(t'_{i+n} - t'_i)\right]. \tag{2}$$

These equations are written for all of the blocks into which the soil is divided. The equations for the blocks adjacent to the boundaries of the region are somewhat different in form. Thus, for the block adjacent to the right-hand boundary, the second term in the brackets assumes the form

$$\frac{2l_i}{\frac{1}{\lambda_i}\ln\frac{r_{i\,\mathrm{amb}}}{\sqrt{0.5(r^2_{ib}+r^2_{i\,\mathrm{amb}})}}}(t^0_i - t'_i).$$

For the block adjacent to the lower boundary, the fourth term in the brackets has the form

$$\frac{2(r^2_{(i+1)b} - r^2_{ib})}{l_i/\lambda_i}(t_l - t'_i).$$

For the block adjacent to the left-hand boundary, the first term is

$$\frac{2l_i}{\frac{1}{\lambda_i}\ln\frac{\sqrt{0.5(r^2_{ib}+r^2_{(i+1)b})}}{r_{ib}}}(t'_{i\,\mathrm{amb}} - t'_i).$$

For the block adjacent to the upper boundary, the third term is written in the form

$$\frac{r^2_{(i+1)b} - r^2_{ib}}{\frac{l_i}{2\lambda_i} + \frac{l_{\mathrm{sn}}}{\lambda_{\mathrm{sn}}} + \frac{1}{\alpha}}(t'_{\mathrm{amb}} - t'_i).$$

Since the Reynolds numbers are usually higher than 10^6 in the flow of a gas through a well, the temperature of the walls of the well are assumed to be equal to the gas temperature. The heat-balance equation for the moving gas (without consideration of variations in the gas temperature as a consequence of the throttling process) has the form

$$Gc(t''_i - t''_{i-n}) = \frac{1}{R_{i-(i+1)}}(t''_i - t'_{i+1}),$$

$$R_{i-(i+1)} = \frac{1}{2\pi\lambda_{i+1}l_i}\ln\frac{\sqrt{0.5(r^2_{(i+1)b}+r^2_{(i+2)b})}}{r_{(i+1)b}}. \tag{3}$$

After the substitutions and the transformations, we write (3) in the form

$$t''_{i-n} = t''_i - \frac{2\pi\lambda_{i+1}l_i}{G(a+bt''_i)\ln\frac{\sqrt{0.5(r^2_{(i+1)b}+r^2_{(i+2)b})}}{r_{(i+1)b}}}(t''_i - t'_{i+1}). \tag{4}$$

We begin the calculations of the wall temperatures at the various levels from the bottom. Here, for the bottom row of blocks we substitute $t''_i = t''_{i+1} = t_l$ into (4), i.e., the wall temperature in this row is found to be equal to the temperature of the gas-bearing layers: $t''_{i-n} = t_l$.

3. Figure 1 shows the block diagram of the program for the solution of this problem on a BÉSM-4 digital computer. The program, together with the information, occupies 2550 cells of the operational memory and is made up of the following blocks:

1) the calculation of the initial conditions t_i^0 of the steady-state thermal regime of the soil on the basis of the formulas of the one-dimensional temperature field for the average annual temperature of the ambient air. Transmission of the derived tabulated values of t_i' and t_i'';

2) calculation of complexes depending on the dimensions of the blocks and contained in (2);

3) transmission of the values of λ_i and c_i;

4) calculation of t_i'' for the wall of the well according to (4);

6, 8, 10, 12, 14, and 16) calculation of t_i'' for the soil blocks according to (2);

5, 7, 9, 11, 13, 15, and 17) transposing t_i'' to the position of t_i' to calculate the next time interval $\Delta\tau$;

18) time counter $\tau = \Sigma\Delta\tau$;

19) determination of the outside temperature t_{amb} and the surface snow cover;

20) determination of the latent heat. If $t_i'' < 0$ when $t_i' \leq 0$ or $t_i'' > 0$ and $t_i' \geq 0$, the calculation is performed in accordance with the usual algorithm involving the use of λ_i and c_i of frozen or thawed soils, respectively. If $t_i'' > 0$ when $t_i' \leq 0$, we compare the quantities of heat required to heat the soil from t_i' to 0 and to thaw out the active water which this soil contains with the quantities of heat reaching the block in $\Delta\tau$ (the part of (2) contained in the brackets). So long as the quantity of incoming heat does not exceed that required, $t_i'' = 0$; as soon as it exceeds that quantity, the calculation is continued in accordance with the usual algorithm, with the use of the λ_i and c_i for the thawed soil. The calculation is performed in analogous fashion when the soil freezes ($t_i' \geq 0$, $t_i'' < 0$).

4. In approximating the linear differential equations to obtain a converging solution, we must satisfy the following relationships between the time ($\Delta\tau$) and space (Δx, Δy) intervals:

$$\Delta\tau \leqslant \frac{1}{2a\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)}. \tag{5}$$

Since in this case we have complex boundary conditions (a jumpwise change in the conditions of heat transfer at the surface of the soil – the deposition and melting of snow, the change in the air temperature) and a change in the coefficient of thermal diffusivity on phase transition, the use of Eq. (5) to determine the magnitude of the time interval $\Delta\tau$ is impossible. We therefore performed certain control calculations with various values for $\Delta\tau$.

Figure 2 shows the process of temperature variation in the smallest block of the calculation scheme (the larger the block, the greater the probability of convergence). Formula (5) yields $\Delta\tau \leq 200$ h for this block; however, we see from the graph that the process converges only when $\Delta\tau \leq 100$ h. The calculation of the annual regime with an interval of $\Delta\tau = 100$ h requires approximately 1.5 min of machine time, which is quite acceptable.

Figure 3 shows the isotherms about the well, calculated on the basis of the following data: the average annual temperature of the ambient air is $t_{amb} = -9.9$°C, the snow cover with an average depth of h = 30 cm remains for 7 months, at a depth of 1300 m the average gas temperature is $t_l = +32$°C, the gas flow rate is G = $776 \cdot 10^3$ kg/day, the well diameter is d = 0.2 m, and the thermophysical properties of the soil vary with depth in the following manner:

Depth, m;	0–200	200–420	420–530	530–640	640–1190	1190–1300
Coefficient of thermal conductivity, W/(m · deg)	2.2	1.5	1.1	1.8	1.0	1.4
Gravimetric heat capacity, kJ/(kg · deg);	0.9	0.84	0.76	0.84	0.76	0.84
Bulk weight, kg/m^3	2000	2200	2600	2600	2600	2600

The solid lines show the temperatures in the soil after τ = 6600 h from the start of the operation of the well, and the dashed lines show the values after 3000 h.

NOTATION

c_i, γ_i are, respectively, the specific heat capacity and the density of the soil in the i-th block;
a is the coefficient of thermal diffusivity for the soil;
l_i is the height of the block;
r_i, r_{iamb} are, respectively, the radius of the inside and the outside circumferences of the block;
t_i'', t_i' are the temperatures at the center of the block, at the end and at the beginning of the calculation time interval $\Delta\tau$, respectively;
t_i^0 is the same, for the steady-state regime, without any thermal effect from the well;
t_{amb} is the temperature of the ambient air;
t_l is the temperature of the layer;
t_{ic}' is the temperature of the wall of the well at the level of the block center;
λ_i is the coefficient of thermal conductivity for the soil in the i-th block;
n is the number of blocks in the horizontal row ($n = 8$);
l_{sn}, λ_{sn} are, respectively, the thickness of the snow cover and the coefficient of thermal conductivity for the snow;
α is the coefficient of heat transfer between the ground surface and the air;
G is the weight flow rate of the gas;
c is the heat capacity of the gas.